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An association between understanding cardinality and analog magnitude representations in preschoolers

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ABSTRACT

The preschool years are a time of great advances in children's numerical thinking, most notably as they master verbal counting. The present research assessed the relation between analog magnitude representations and cardinal number knowledge in preschool-aged children to ask two questions: (1) Is there a relationship between acuity in the analog magnitude system and cardinality proficiency? (2) Can evidence of the analog magnitude system be found within mappings of number words children have not successfully mastered? To address the first question, Study 1 asked three- to five-year-old children to discriminate side-by-side dot arrays with varying differences in numerical ratio, as well as to complete an assessment of cardinality. Consistent with the analog magnitude system, children became less accurate at discriminating dot arrays as the ratio between the two numbers approached one. Further, contrary to prior work with preschoolers, a significant correlation was found between cardinal number knowledge and non-symbolic numerical discrimination. Study 2 aimed to look for evidence of the analog magnitude system in mappings to the words in preschoolers' verbal counting list. Based on a modified give-a-number task (Wynn, 1990, 1992), three- to five-year-old children were asked to give quantities between 1 and 10 as many times as possible in order to assess analog magnitude variability within their developing cardinality understanding. In this task, even children who have not yet induced the cardinality principle showed signs of analog representations in their understanding of the verbal count list. Implications for the contribution of analog magnitude representations towards mastery of the verbal count list are discussed in light of the present work.

1. Introduction

Learning to count meaningfully is a slow and arduous process for young children, typically taking several years to master. Around the second birthday children begin to produce memorized count lists, eventually assigning number words to a set of items in a stable order with one-to-one correspondence, one word for each item. Children then begin to learn the exact meanings for small numbers, such that "one" refers to exactly one item, "two" refers to exactly two items, and so on, but only after much experience and time has passed do children fully grasp the principle of cardinality, a key step in the acquisition of the counting principles whereby children understand that each number in the count list maps to a single quantity (e.g., Fuson, 1988, 1992; Le Corre, Brannon, Van de Walle, & Carey & Sarnecka, 2006; Schaeffer, Eggleston, & Scott, 1974; Wynn, 1990, 1992).

Ongoing debate in the early number development literature focuses on understanding the prerequisites necessary for learning to count, and early work by Wynn

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(1992) comments on the importance of such an approach: "How humans come to learn about the counting system of their culture is closely related to the nature of our initial representation of number, because in order to understand counting we must somehow relate it to our prior number concepts" (p. 220). Growing evidence has identified two basic systems for tracking quantity information in humans and non-human animals, the object-individuation system and the analog magnitude system (for a review, see Feigenson, Dehaene, & Spelke, 2004), and both systems have been offered as early foundations of later numerical concepts, such as counting (e.g., Carey, 2001, 2004; Dehaene, 2001; Gelman & Gallistel, 2004; Le Corre & Carey, 2007).

The object-individuation system has a limited capacity and is able to track sets containing up to 3 to 5 items (e.g., Feigenson, Carey, & Hauser, 2002; Hauser, Carey, & Hauser, 2000; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). As described by Carey (2001) and others, in this system"…number is only implicity encoded," with a generic placeholder given to each item being tracked. For example, two items might be represented as 'item item', three as 'item item item', but there is no inherent coding of quantity relationships in these representations (e.g., Carey, 2001; Le Corre & Carey, 2007).

The analog magnitude system, on the other hand, is a noisy representational system that explicitly codes for quantity, with the mental representation of a set of items proportional to the number of items being represented. This system treats discrete quantities (e.g., three items) as analogous to continuous magnitudes (e.g., a line of a certain length) and due to increasing variability as the quantity represented increases, this system operates as a function of Weber's law, whereby two quantities become more difficult to differentiate as they become closer together (e.g., Brannon & Terrace, 1998, 2000; Cantlon & Brannon, 2006; Halberda & Feigenson, 2008; Jordan & Brannon, 2006; Meck & Church, 1983; Xu & Spelke, 2000). Because the analog magnitude system is powerful enough to represent all quantities, both large and small, some researchers posit that this system might play an important role in the development of more complex numerical concepts (Dehaene, 1997, 2001; Gallistel & Gelman, 1992, 2000; Gelman & Gallistel, 1978, 2004; Wynn, 1992, 1998).

Past research has examined the link between analog magnitude representations and mastery of the verbal count list using two approaches: One set of studies looked for an association between analog magnitude representations and counting ability (Huntley-Fenner & Cannon, 2000; Rousselle, Palmers, & Noel, 2004; Slaughter, Kamppi, & Paynter, 2006), while the second set of studies looked for evidence of analog magnitude representations in children's mapping of the words in their verbal count list (Le Corre & Carey, 2007; Sarnecka & Lee, 2009). While no study has found clear evidence for a relation between these two numerical skills, closer examination of these studies reveals some limitations that undermine strong conclusions about the link (or lack thereof) between analog magnitude representations and verbal counting proficiency.

Work by Huntley-Fenner and Cannon (2000) and Slaughter et al. (2006) tested the analog magnitude system in preschoolers using numerosity comparison tasks and found no correlations with counting. Researchers asked children to compare two sets of objects and found that quantities became harder to discriminate as the ratio between the objects became more difficult, a phenomenon known as the distance effect. Importantly, if discriminability varied in accord with Weber's law as expected by the analog magnitude system, performance would decline as a function of ratio, not absolute set size, and this was not the case in either study: In both studies, children showed significantly decreasing accuracy as the comparison numbers increased. For example, Huntley-Fenner and Cannon (2000) found that children were less accurate at discriminating 5 vs. 10 than 3 vs. 6; Slaughter et al. (2006) have demonstrated that younger children were less accurate at discriminating 10 vs. 20 than 3 vs. 6. If children's performance was based on ratio, as expected by the analog magnitude system, this pattern is not only unpredicted, but also inconsistent with both infant work (e.g., Xu & Spelke, 2000) and adult work (e.g., Barth, Kanwisher, & Spelke, 2003).

Huntley-Fenner and Cannon (2000) and Slaughter et al. (2006) concurrently tested children's counting proficiency, but these tasks also contained limitations. Both studies gave children sets of varying sizes and asked children "How many *x* are there." Huntley-Fenner and Cannon (2000) tested children's ability to correctly count sets of up to 15 blocks; Slaughter et al. (2006) tested their ability to correctly count sets of up to 30 stickers. In both cases, the highest correct tag was taken as a measure of the child's counting ability. No associations were found between this tag and performance on the magnitude comparison tasks, and these researchers argued that magnitude-based discriminations are, therefore, independent of verbal counting ability in preschoolers.

Yet based on the research of Fuson and others (e.g., Fuson, 1988; Fuson & Hall, 1983; Schaeffer et al., 1974; Wynn, 1990, 1992), we know that verbal counting develops over a protracted period of time that culminates in the understanding of cardinality. The ability to reliably report the final tag of a count in answer to the question "How many?" is an earlier achievement which may not reflect the heart of children's numerical representations, and need not reflect an understanding of cardinality at all.

Rousselle et al. (2004) attempted to examine the relation between analog magnitude representations and counting skills, including cardinality. This work used a magnitude comparison task with various controls for continuous variables (e.g., surface area, density), and performance on a test of cardinal number knowledge (e.g., children asked to "give the experimenter *x* items"). The results were varied. Some evidence did point to an association between magnitude comparison skills and cardinality understanding (i.e., children with some number knowledge were better at magnitude comparisons than children with no number knowledge), but both the magnitude comparison task and the cardinality task leave open methodological questions which limit conclusions from this work. First, the magnitude comparison task found no evidence that 3-year-olds used the analog system for discrimination when surface area was controlled, a finding not supported by other work with this age group (e.g., Cantlon, Safford, & Brannon, 2010; Halberda & Feigenson, 2008). Second, the validity of the cardinality task used is unclear, as the cardinality of "one" was never assessed, and children appeared to answer each cardinality request only once, making this measure less reliable than previous cardinality assessments (see Wynn, 1990, 1992).

More recent work by Le Corre and Carey (2007) took a more direct look at the relation between analog magnitude representations and the words in the verbal count list in preschoolers. Three- and 4-year-olds were asked to produce estimates of quickly presented dot arrays ranging from 1–10 to test for the analog system, and performed Wynn's (1990, 1992) give-a-number task to assess cardinal number knowledge for the numbers 1–6. These researchers hypothesized that if analog magnitude representations play a role in helping children understand the meanings of the words in their count list, preschoolers who had not yet mastered cardinality would show evidence of the analog magnitude system during the dot estimation task (i.e., numerical estimates would become noisier as the number of items increased, known as the size effect) for numbers above their level of cardinality understanding. Through a series of analyses, this study found that children who had partial knowledge of the cardinality principle (classified as 1-, 2-, 3-, and 4-knowers by the authors) showed no evidence of increasing numerical estimates for numbers in their count list, therefore, showing no understanding that larger quantities are represented by larger number words (Le Corre & Carey, 2007). A group of children who had mastered the cardinality principle did show this mapping of larger number words onto larger quantities, and Le Corre and Carey (2007) concluded that children only map the meaning of the words in their count list to the analog magnitude system *after* they have induced the cardinality principle. Therefore, they reasoned, it must not be involved in the acquisition of the counting principles.

One important methodological limitation from the work of Le Corre and Carey (2007) comes from the use of an estimation task which required verbal responses in children, as it is possible that the mapping of analog magnitude representations to words in a child's count list could be better measured using a verbal comprehension task that induced enough trials to measure variability. Work by Sarnecka and Lee (2009) took a step towards such an approach, examining children's errors during Wynn's (1990, 1992) give-a-number task, a measure requiring only verbal comprehension. This work found no evidence of increasingly variable estimates in the errors children made during this task, but several limitations make this result inconclusive with regard to the analog magnitude system. First, the datasets re-analyzed by Sarnecka and Lee (2009) asked children to respond to an incomplete set of values between 1–10, and only a subset of children were ever asked to give-a-number for values beyond four or five. This prior work, therefore, did not have sufficient range and repetition to systematically evaluate how variability in responses on a cardinality task might relate to the analog magnitude system. Further, traditional work examining response variability for evidence of the analog magnitude system analyzes an individual's mean response, standard deviation of that response, and coefficient of variation (COV; ratio of standard deviation to mean estimate), none of which were calculated by Sarnecka and Lee (2009).

A recent study by Opfer, Thompson, and Furlong (2010) had preschoolers perform a variation of the give-a-number task, asking once for each of the numbers 1–9. This work found evidence of analog magnitude representations in the mappings of these words to the verbal count list, with both mean response and standard deviation increasing as the number requested increases. However, because the aim of Opfer et al. (2010) was to examine groups of preschoolers who had or had not made spatial-numeric mappings, testing each individual participant's cardinality proficiency was beyond the scope of that study, and therefore, it cannot be ruled out that some of the children who showed this mapping to the number words may have already induced the cardinality principle, consistent with the findings of Le Corre and Carey (2007).

In summary, prior studies examining the role analog magnitude representations in mastery of the verbal count list have found no relation between these skills (or a limited one, as in the case of Rousselle et al., 2004). Studies which looked for a correlation between these skills were limited by poor measures of both the analog magnitude system and counting proficiency (Huntley-Fenner & Cannon, 2000; Rousselle et al., 2004; Slaughter et al., 2006), and studies which looked for evidence of mappings from the analog magnitude system to the verbal count list were limited because of verbal task demands imposed on preschool participants (Le Corre & Carey, 2007) and insufficient design to systematically measure the analog system in estimations (Sarnecka & Lee, 2009).

The present set of studies aimed to first assess the correlation between the analog magnitude system in preschoolers and their cardinality understanding, and second to look for evidence of analog magnitude representations within preschoolers' mappings of the words in their count list. In Study 1, preschoolers performed a numerosity comparison task and a test of cardinality, and the association was examined. Study 2 then expanded the test of cardinality used in Study 1 and provided an opportunity for children to produce enough errors for analysis of the COV directly from the measure used to assess the cardinality principle, a measure that required only verbal comprehension of the number words, as opposed to verbal production.

2. Study 1

2.1. Method

2.1.1. Participants

Forty-eight 3- to 5-year-old children (mean age 3;11, range 3;0 to 5;3) participated. Children were tested either at a local preschool or a university child development laboratory. Twenty-eight additional children were invited to participate, but were ultimately excluded from the study for being unable to complete the numerical comparison task, either because they never fully understood the instructions (8), they stopped following the instructions midway (16), or they failed to perform the cardinality task (4). One child had participated in an earlier version of the study and was also excluded.

2.1.2. Design

Each child participated in two tasks, a numerosity comparison task and a cardinality task.

The numerosity comparison task was designed with two within-subjects variables: number ratio and set size. Number ratio had three levels: 1:2, 2:3, and 3:4. For each number ratio tested, there were four set sizes tested. For example, the number ratio 1:2 had the following four dot array comparisons: 1 vs. 2, 4 vs. 8, 15 vs. 30, and 25 vs. 50. Table 1 shows all comparisons used.

Mastery of the verbal count list was assessed in the same session with a test of cardinality in which children were asked to put a certain number of objects into a box, based on Wynn's give-a-number task (1990, 1992).

2.1.3. Materials

The stimuli used for the numerosity comparison task were dot arrays ranging in numbers from 1–50. Paired arrays appeared on a Macintosh PowerBook running the program PsyScope. A trial presentation consisted of two pictures side-by-side, each measuring 5.25 in. × 6.5 in. and separated by a distance of 0.5 in. Each picture contained black dots randomly dispersed over a 10 × 13 unit grid on a white background.

Past studies have pointed to area as a highly salient cue for quantity discrimination (e.g., Feigenson, Carey, & Spelke, 2002), and work with preschoolers by Rousselle et al. (2004) found that children were *unable* to successfully compare quantities when the displays were controlled for surface area (and children had no such difficulty for stimuli controlled for density, contour length, and item size). In order to prevent preschoolers from basing their judgments on the continuous dimension of area rather than number, the dot arrays were matched for total filled area. For example, if the comparison was 25 vs. 50 dots, one picture would contain 25 dots while the other contained 50 dots which were half as large. Fig. 1 shows a sample stimulus pair.

For the cardinality task, the stimuli included a cardboard box and 12 plastic fish.

2.1.4. Procedure

Each child sat at a table in front of the laptop while the experimenter sat next to the child. Before the first practice

Table 1
The 12 numerosity comparisons used in Study 1; each ratio tested had four set sizes.

Ratio	Set size			
	1	2	3	4
1:2	1 vs. 2	4 vs. 8	15 vs. 30	25 vs. 50
2:3	2 vs. 3	6 vs. 9	16 vs. 24	30 vs. 45
3:4	3 vs. 4	9 vs. 12	18 vs. 24	30 vs. 40

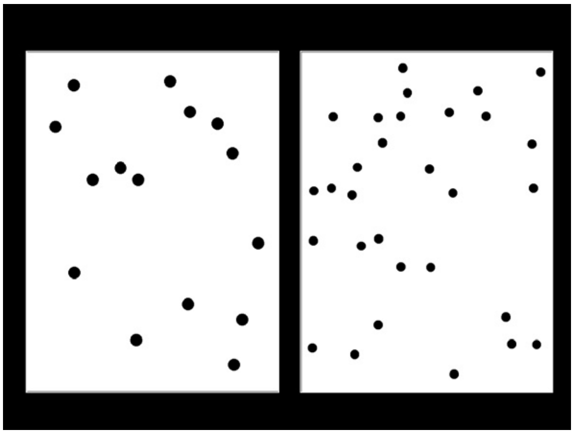

Fig. 1. Sample stimuli used in Study 1. Dot arrays were presented side-by-side and children were asked to point to the side with more dots.

trial was presented, the experimenter explained the following as the children looked at a black screen with a small white crosshair in the center: "I'm going to show you two pictures with dots on them (gesturing to the corresponding locations on the laptop screen), and I want you to point to the side that has more dots." The experimenter pressed the spacebar to start a trial. Children received two practice trials with pairs remaining on the screen for five seconds. The first pair compared 1 vs. 2 and the second pair compared 6 vs. 12. After five seconds, the pair of arrays disappeared and the screen went black. If the child pointed to the left, the experimenter pressed the "s" key; if the child pointed to the right, the experimenter pressed the "d" key. The fixation point then reappeared. Children were told whether they were correct or incorrect.

After completing these "slow" practice trials, children were told: "Now we are going to do the same thing, but this time the pictures are going to come up and go away really fast (gesturing as above), but I want you to do the same thing and point to the side that has more dots." Four "fast" practice trials with a shortened display time of 750 ms were then presented to the child. This brief presentation time aimed to limit alternative counting-related strategies. Two of these trials were again 1 vs. 2 and the other two trials were 6 vs. 12. The higher number appeared one time on the right and one time on the left for each of these pairs.

After finishing all six practice trials, test trials began. Each child received two blocks of 12 trials in one of four orders, with a break in between the two blocks.

During the break in the numerosity comparison task between the two blocks, the children participated in the first half of the cardinality task. For this, the twelve fish were placed on the table. The child was then asked "Can you put *x* fish in the box?" where *x* varied from 1 to 7, in a randomized order until all seven cardinalities were tested. Neutral feedback was given after each trial.

Once all seven cardinalities were completed, children were reminded of the rules and objective of the numerosity comparison game and given the second block of 12 trials.

After completing these trials, children were asked to perform the cardinality task a second time, placing each set size from 1 to 7 into the box, this time in a different randomized order.

2.2. Results

2.2.1. Numerosity comparison task

Children's accuracy in choosing the larger of the two arrays was calculated. An omnibus analysis of variance (ANOVA) revealed no effects of gender, order, or age. Therefore, subsequent analyses were collapsed over these variables.

The first thing to note is that children's performance on this task was significantly better than that predicted by chance (50%). They averaged 63% accuracy in identifying which side had more dots ($SD = 13\%$), $t(47) = 6.682$, $p < .0001$. When broken down into each of the three individual ratio levels as well as the four set size levels, performance remained above chance (all t's > 2.9 and all p's < .005).

To test whether children's performance decreased as the ratio between the number pairs approached one, we performed a 3 × 4 repeated-measures analysis of variance (ANOVA) with ratio (1:2, 2:3, 3:4), and set size (level 1, 2, 3, 4) as variables. We found that children's performance varied significantly with ratio, $F(2, 94) = 9.090$, $p < .0001$, eta = .162 (see Fig. 2). Contrast analyses revealed that children's accuracy was significantly higher at the 1:2 ratio than the 2:3 ratio ($F(1, 47) = 12.864$, $p < .001$, eta = .215), but that children's accuracy at the 2:3 and 3:4 ratios did not differ. No main effect of set size or interaction between number ratio and set size was observed (p's > .20). Fig. 3 shows accuracy for each number ratio across the four set sizes.

2.2.2. Cardinality task

Three scores were derived from children's cardinality performances; overall accuracy, the highest correct response made in either of the two blocks, and the highest correct response made in both blocks. A highest correct response did not guarantee that the child made no mistakes

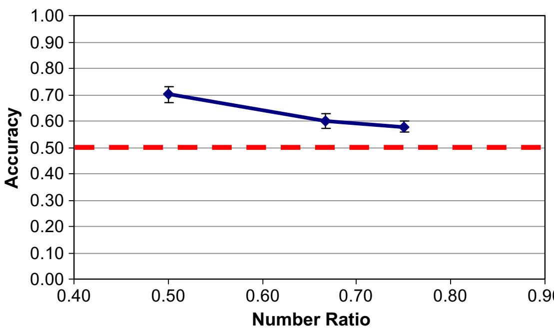

Fig. 2. Performance on the numerosity comparison task across number ratio. A main effect of number ratio was found ($F(2, 94) = 9.090$, $p < .0001$), with higher accuracy for comparisons in a 1:2 (.5) ratio than a 2:3 (.67) or 3:4 (.75) number ratio. No other effects were found. Accuracy for each number ratio level was significantly different from chance (50%) (all t's > 3.7, all p's < .001).

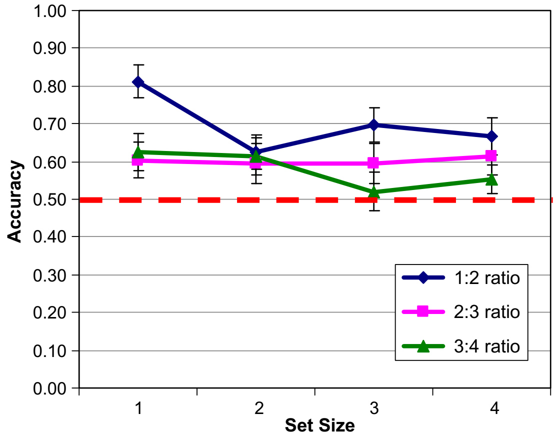

Fig. 3. Performance on the numerosity comparison task across all number ratio and set size levels. There is no main effect of set size, $F(3, 141) = 1.56$, $p = .20$, and no interaction between number ratio and set size, $F(6, 282) = 1.25$, $p = .28$.

at lower cardinalities, as in the case of a child whose highest response on both blocks was "7", yet overall only achieved 93% accuracy.

2.2.2.1. Accuracy. Overall children were correct on 72 % ($SD = 29\%$) of the cardinality trials, with a minimum of 14% and a maximum of 100%. Fifteen children got all 14 trials correct.

2.2.2.2. Single highest correct response. The average single highest correct response was 5.4 ($SD = 2.03$). Two children had a highest response of 1, six children had a highest response of 2, two children reached 3 at least once, four children reached 4, seven children reached 5, two children reached 6, and 25 children reached 7.

2.2.2.3. Consistently highest correct response. Six children performed only one round of the cardinality task and are not included in this analysis. For the remaining 42 children, the average highest correct response achieved on both trials was 4.7 ($SD = 2.23$). Three children correctly responded with 1 twice. Eight children correctly responded with 2 twice. Four children responded 3 twice. Three children responded 4 twice. Four children responded 5 twice. Four children responded 6 twice, and sixteen children correctly responded 7 on both trials.

As expected, the three scores were all highly correlated with each other, with all r^2 values greater than .9. All three were also correlated with age, with r^2 between .62 and .74.

2.2.3. Comparisons across number tasks

In order to test whether there is an association between precision in a child's analog magnitude system and their acquisition of verbal counting skills, we calculated correlations between children's overall accuracy on the magnitude comparison task and the three measures of cardinality proficiency After partialling out the influence of age, accuracy on the magnitude comparison task significantly correlated with overall accuracy on the cardinality

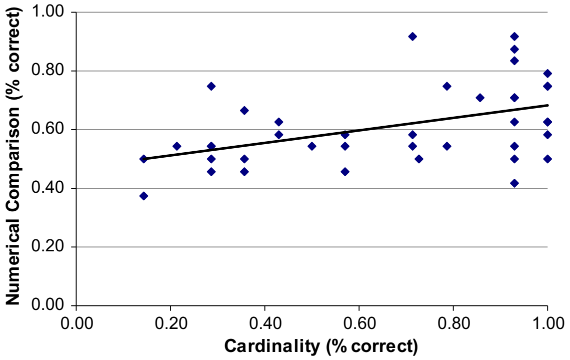

Fig. 4. After partialling out age, there was a significant correlation between overall accuracy on the magnitude discrimination task and cardinality accuracy ($r^2 = .422$, $p < .006$). Similar correlations with highest single correct cardinality ($r^2 = .405$, $p < .01$) and highest correct cardinality twice ($r^2 = .404$, $p < .01$) were also found.

task (see Fig. 4), highest correct cardinality response, and highest correct cardinality twice (all r^2's > .40, all p's < .01).

2.2.4. Summary

During the numerosity comparison task used in the present study, evidence of the analog magnitude system was found, with accuracy modulated by the ratio between the quantities being compared. Importantly, this held true across set size. Although past work (e.g., Halberda & Feigenson, 2008) finds evidence of better discrimination of items in a 2:3 ratio as compared to a 3:4 ratio in children at this age, the brief presentation time (750 ms) used in the present experiment (as compared to 1200–2500 ms in Halberda & Feigenson) made it difficult to replicate this finding.

This measure of the analog magnitude system was then correlated with cardinal number knowledge, showing a significant relation between performance on these two tasks, contrary to prior work with preschoolers. This correlation contributes to understanding how the analog magnitude system might underlie mastery of the verbal count list, though it is important to recognize the possibility that a third variable could be mediating this relationship. Further, it is important to note that the measure of the analog magnitude system of number representation was limited by the choice to control stimulus displays only on the continuous dimension of area. Children could have adopted a strategy of choosing the side with smaller dots as the more numerous side in order to show evidence of the distance effect, and future work exploring the analog magnitude system will control for such a possibility.

3. Study 2

In Study 1 we found a significant positive association between preschooler's analog magnitude representations and cardinality proficiency, consistent with theories emphasizing the role of analog magnitude representations in the development of more advanced numerical concepts (e.g., Dehaene, 1997, 2001; Gallistel & Gelman, 1992, 2000; Gelman & Gallistel, 1978, 2004; Wynn, 1992, 1998). Specifically, the more accurate a child was at discriminating dot arrays varying in ratio, the more proficient their cardinal number knowledge was, independent of age.

This correlation is an important first step in assessing the association between cardinality development and the analog magnitude system, but like prior work, this result provides only an indirect link between these skills. However, further examination of the modified version of Wynn's (1990, 1992) give-a-number cardinality task used in Study 1 revealed the potential for a more direct look at the relation between these numerical abilities. In prior research, the give-a-number task has been used primarily to determine cardinal number knowledge in preschoolers and to compare this to performance on other tasks (e.g., Condry & Spelke, 2008; Le Corre et al., 2006; Le Corre & Carey, 2007; Wynn, 1990, 1992). Using the titration method developed by Wynn (1990), children are first asked to give 1 item, and if successful they are then asked for 2, then 3, and so on (with a typical limit at 6 items). If children fail to give the correct response for any number, x, they are then asked again for $x - 1$, and if successful at $x - 1$, will be asked for x again. The number at which children are at least 66% accurate is then referred to as their "knower-level." Children who are 66% accurate for all quantities requested, are said to have induced the cardinality principle ("CP-knowers").

A preliminary analysis of the variability in responses produced by children in Study 1 (and critical review of the work of Sarnecka & Lee, 2009) suggested that more trials across a wider range of numbers could potentially induce sufficient errors for calculating the role of the analog system in a cardinality task. Specifically, with increased variability, it would be possible to calculate the COV, a measure that has been used during estimation tasks with adults and older children to look for evidence of the analog magnitude system (e.g., Cordes, Gallistel, Gelman, & Whalen, 2001; Huntley-Fenner, 2001; Whalen, Gallistel, & Gelman, 1999).

The present study (Study 2) extended the cardinality task of Study 1 in order to investigate the variability in children's responses when prompted to give 1–10 items repeatedly. Using the standard deviation and mean response for a given target, the goal was to look at COV scores for children who had not yet mastered cardinality. We predicted that, if the analog magnitude system contributes to cardinality development, children who have not yet induced the cardinality principle will show evidence of scalar variability for cardinalities they have not fully mastered, with a constant ratio between the standard deviation and mean response for each number requested. Importantly, this task required no verbal responses from children, minimizing task demands and capturing a more naturalistic response to requests for numbers outside of their cardinality proficiency.

3.1. Method

3.1.1. Participants

Thirty-nine 3- to 5-year-old children (mean age 4;1, range 3;0 to 5;4) participated. Children were tested at a university child development laboratory. Thirty-seven additional children were invited to participate, but were ultimately excluded from the study for performing above 66% accuracy when probed for 10 items and/or counting aloud during responses (21), failing to perform a minimum number of trials (15), or never fully understanding the instructions due to English as a second language (1). One child failed to respond independently during the task, instead attempting to rely on experimenter cues, and was, therefore, excluded as well.

3.1.2. Design

Children performed a cardinality task similar to that used in Study 1. As compared to Study 1, this test of cardinality extended the range of quantities tested, the items used, and the number of repetitions requested.

3.1.3. Materials

The materials for this task included seven unique sets of toys (e.g., 30 small orange fish; see Table A in Appendix A for a list of stimuli used) and a set of plastic boxes.

3.1.4. Procedure

Each child sat at a small table across from the experimenter. The child was told that he/she would be playing games with some different toys. The first set of stimuli, always consisting of 30 small orange fish, was then emptied into a pile in front of the child. A plastic container was then placed on the table, and the child was told "This is a pond where the fish like to go swimming." The child was asked "Can you put *x* fish in the pond?" where *x* varied from 1 to 10 in a randomized order until all ten cardinalities were tested. After these ten trials, children were presented with a new set of stimuli and once again asked to put varying quantities of items into a container. The order of subsequent sets of stimuli followed the same order for all children, while the order of the requested numbers (1–10) was randomized differently for each set. Children continued to receive new sets of stimuli for this cardinality task until they expressed a verbal interest in stopping or no longer would respond to the experimenter's requests. Only neutral feedback was given throughout the task.

3.2. Results

For all children in the present study, average response to each target request was calculated. On a subset of trials, preschoolers responded by placing all items in the appropriate container, an action that could be interpreted in several ways. This response could represent an estimation of "a lot" for numbers deemed large, or it could represent a perseverative error, whereby children begin putting in "*x*" items one-by-one and do not stop until there are no more items left. Because of the difficulty in interpreting "all" responses, data were analyzed both with and without such trials. In the present paper, analyses are presented without "all" trials; however, Figs. C1–C3 in Appendix C present findings obtained with the inclusion of trials where children responded with "all."

While past work using the give-a-number task typically identifies 4–5 categories of cardinality understanding using the titration method (e.g., 1-, 2-, 3-, 4-, and CP-knower), proficiency in the present study was operationalized as the highest number at which a child was 66% accurate. Table 2 displays a summary of the children in Study 2 broken down by proficiency level (p), illustrating the number of children at a given level (n), as well as mean values for age in months, number of trials, overall accuracy during the task, and the highest correct response overall.

3.2.1. Scalar variability in children with partial cardinal number knowledge

Past research using the give-a-number task identifies children as CP-knowers if they respond to prompts of five or more items with 66% accuracy (e.g., Le Corre & Carey, 2007; Le Corre et al., 2006; Sarnecka & Lee, 2009). Thus, in order to be conservative, only the performance of children below a proficiency level of five was analyzed to determine whether children without knowledge of the cardinality principle show systematic variability within their cardinality responses. The following analyses examine the 24 children with proficiency levels between 1 and 4.

Three pieces of evidence are used to document a size effect in an estimation task: (1) mean estimates that increase as the target value increases; (2) standard deviations of mean estimates that increase as the target value increases; and (3) a constant COV. Fig. 5 illustrates that across all children with proficiency levels 1–4, as the

Table 2
Summary of children separated by proficiency level (cardinality level with 66% accuracy).

Proficiency level (p)	Number of children (n)	Age (*SD*)	Cardinality task		
			Number of trials	Overall accuracy (*SD*)	Highest correct response
One	2	44.50 (*0.71*)	37.5	0.28 (*0.10*)	9.50
Two	7	44.71 (*6.02*)	48.6	0.33 (*0.03*)	7.29
Three	9	44.44 (*5.75*)	58.3	0.37 (*0.04*)	7.00
Four	6	50.33 (*8.31*)	60.0	0.55 (*0.09*)	8.83
Five	7	54.86 (*7.01*)	60.0	0.61 (*0.06*)	9.71
Six	3	48.67 (*3.06*)	60.0	0.57 (*0.03*)	9.67
Seven	3	54.00 (*8.00*)	56.7	0.73 (*0.07*)	10.00
Eight	1	47.00	62.0	0.69	10.00
Nine	1	52.00	30.0	0.88	10.00

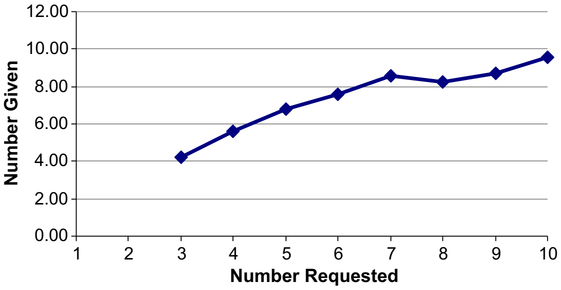

Fig. 5. Mean number of items given collapsed across values beyond a child's proficiency level for requests of 3–10 items. The average slope of the mean response for items beyond a child's proficiency level is positive and significantly different from 0, $p = 0.001$.

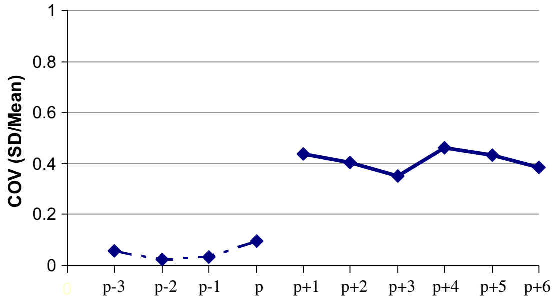

Fig. 7. Coefficient of variation values collapsed across proficiency levels (p) of 1, 2, 3, and 4 for values from $p - 3$ through $p + 7$. The COV values beyond p show a slope no different from 0, $p > 0.75$. Further, the COV values at and below proficiency also show a slope no different from 0, $p > 0.50$.

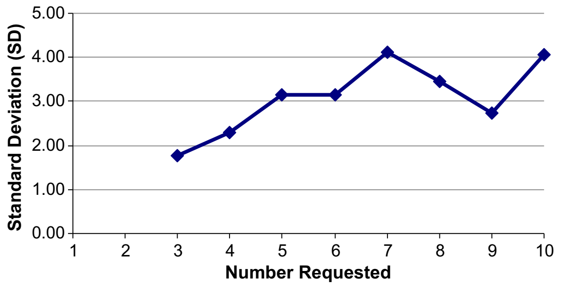

Fig. 6. Standard deviation collapsed across values beyond a child's proficiency level for requests of 3–10 items. The average slope of the standard deviation of responses for items beyond a child's proficiency level is positive and marginally different from 0, $p = 0.06$.

number requested increased, the number given by the child increased as well. For each child, the slope of mean response to requests beyond their proficiency level was calculated, and the mean slope was tested against a line with a slope of 0; the mean slope of 0.61 (*0.75*) was significantly different from 0, $t(23) = 4.03$, $p = 0.001$. Further, each child's slope of the standard deviations for responses to requests beyond their proficiency level was also calculated, and the mean slope was tested against a line with a slope of 0. Fig. 6 shows the mean slope of 0.17 (*0.41*), a value that was marginally different from 0, $t(23) = 2.00$ $p = 0.057$.

For children with proficiency levels between 1 and 4, analyses of mean response and standard deviation show both values increasing as the number of items requested increases. In the same group of children, the mean response and standard deviation values were next used to calculate a set of COV values (*SD*/mean) for numbers below, at, and beyond a child's proficiency level, p (see Fig. 7). Since standard deviation values at and below p are close to 0 (due to minimal errors within a child's proficiency level), COV values at and below p are also expected to be close to 0, as they are calculated using *SD* as the numerator. In the present sample, the average COV value for children at and below p is 0.05. Further, for each of the 22 2-, 3- and 4-proficient children, the slope of COV values below and at proficiency level was calculated (the two 1-proficient children could not have a slope calculated, as they only had a single COV value at p). The average slope of 0.017 was found not to be significantly different from a line of slope 0, $t(21) = 0.65$, $p = 0.52$.

Beyond p, COV values should also be constant with a slope near 0 if children show proportionate increases in their standard deviations and mean responses as the target number increases. The slope for COV values beyond p (the line formed by $p + 1$, $p + 2$, $p + 3$, $p + 4$, $p + 5$, and $p + 6$) was tested against a line with slope of 0, and the mean slope of -0.002 (*0.04*) was not significantly different from 0, $t(23) = -0.30$, $p = 0.77$. Further, individual subjects show slopes with an absolute value ranging from 0.008 to 0.075, illustrating that a constant slope of COV values beyond p is close to 0 for all subjects. These constant COV values found beyond p provide strong evidence of scalar variability in children's estimates of values beyond their proficiency level (see Fig. B and Table B in Appendix B for graphs and table of COV for proficiency levels 2, 3, and 4 individually).

3.2.2. Individual participant's data

Past work has suggested that COV values differ between adults and children, being in the range of .12 to .19 for adults (Whalen et al., 1999) and .11 to .37 for 5–7-year-olds (Huntley-Fenner, 2001). Whether this difference is due to age *per se* or more number-specific mechanisms is unclear. In Study 1, a significant positive correlation was found between precision in the analog magnitude system as measured by a numerosity comparison task and preschoolers' cardinality proficiency, independent of age. In the present study, a parallel analysis examined the associations between age, cardinality proficiency, and mappings of the analog magnitude system to count words beyond a child's proficiency level, p.

An average COV value was calculated for each of the 39 participants using the COV scores for all numbers beyond p. In the present study, preschoolers showed a wide range of COV values, varying from .10 to .69 ($M = .35$, $SD = .14$). Based on past work, lower COV values in the present analysis can be taken as a proxy for higher precision in the mappings of the analog magnitude system to the verbal count list.

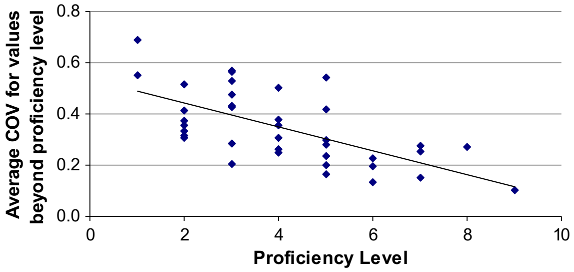

Fig. 8. After partialling out the influence of a child's age, a significant negative correlation is found between a child's cardinality proficiency level and their average COV for values beyond their proficiency level ($p < 0.001$).

Initial analyses showed significant negative correlations between average COV and age, $r^2 = -0.45$, $p < .005$, as well as average COV and proficiency level, $r^2 = -0.65$, $p < 0.001$. Importantly, the correlation between average COV and proficiency level remained significant after partialling out the influence of age, $r^2 = -0.57$, $p < 0.001$; however, after partialling out proficiency level, the correlation between average COV and age was no longer significant, $r^2 = -0.26$, $p = 0.12$. These results show that independent of age, higher precision in the mappings of the analog magnitude system to the verbal count list is significantly correlated with cardinality proficiency (see Fig. 8).

4. General discussion

4.1. Summary of results

Study 1 tested preschoolers on a non-symbolic numerosity comparison task and a cardinality assessment based on Wynn's give-a-number task (1990, 1992). Strong evidence of the distance effect was found in the numerosity comparison task, with children becoming less accurate to report which side had more dots as the ratio between the two arrays approached 1. The relation between performance on the numerosity comparison task and children's performance on a test of cardinality was found to be significantly correlated, independent of age, providing evidence that these number skills are related, in contrast to the conclusions of prior work in this area (e.g., Huntley-Fenner & Cannon, 2000; Le Corre & Carey, 2007; Slaughter et al., 2006). Limitations to the conclusions from this correlational finding come from the possibility of mediation by a third variable other than age.

Extending the findings of Study 1, Study 2 used multiple repeated trials within a cardinality task to generate enough errors to test for variability in mappings of words in the verbal count list in children who have not mastered the cardinality principle. For children who reached their highest cardinality proficiency (66% accuracy) for 1, 2, 3, or 4 items, cardinality responses to values beyond this proficiency level showed systematic variability through a constant COV, with mean response and standard deviation increasing proportionally as the target number increased. Thus, Study 2 provides the first evidence that children who have not mastered the cardinal word principle ("non-CP-knowers") show the signatures of scalar variability directly *within* their mappings of number words beyond their proficiency level.

Further, parallel to Study 1, Study 2 found a significant correlation between a child's average COV score beyond p, taken as a proxy for precision in the mapping of analog magnitude representations to the verbal count list, and cardinality proficiency, independent of age. Prior work with infants and children has found that the precision of these representations, as measured in an independent task, increases alongside increases in a child's age (e.g., Halberda & Feigenson, 2008; Huntley-Fenner, 2001; Lipton & Spelke, 2003; Xu & Arriaga, 2007), but the present work reveals a significant association between the precision in the analog magnitude system mappings to the verbal count list and cardinality proficiency, independent of a child's age.

Study 2 modified the titration method typically used with Wynn's (1990, 1992) give-a-number task by requesting the numbers 1–10 in randomized order. This method undoubtedly increased the task demands on the child and was done deliberately to give children the opportunity to make errors across the entire range of numbers. One consequence of this strategy, however, was to identify individual children who almost certainly *had* induced the cardinality principle, yet produced errors at the higher numbers (thus appearing as 5-, 6-, 7-, 8- or 9-proficient). Because the logic of our argument depends on the variability produced by children who have not yet induced the cardinality principle, and because others have never identified non-CP-knowers with such high levels of proficiency (Le Corre & Carey, 2007; Wynn, 1990, 1992), we conservatively excluded these children from our analysis. Study 2 found clear evidence of the analog magnitude system in mappings of large number words in 1- to 4-proficient children, with increasing estimates as the number of items requested increased, as well as a constant COV for numbers beyond p (see also Appendix B).

4.2. Analog magnitude representations are mapped onto large number words before children master the cardinality principle

The acquisition of the counting principles occurs over a protracted period of time in preschool-aged children, with the concept of cardinality representing a key insight at the heart of this acquisition process. Until now, the strongest research findings have concluded that despite early evidence of the analog magnitude system in infancy (e.g., Xu & Spelke, 2000), this system plays no role in the acquisition of the counting principles; instead, these noisy number representations are said to be mapped onto the verbal counting list only after children have mastered cardinality (Le Corre & Carey, 2007). However, by using a slight variation on the standard methods, we found that children who had not yet mastered the principle of cardinality do show a substantial influence of analog magnitude representations in their mappings of number words beyond their cardinality proficiency. This finding supports prior arguments that this system could lay important foundations in the acquisition of the verbal count list (Dehaene, 1997, 2001; Gallistel & Gelman, 1992, 2000; Gelman & Gallistel, 1978, 2004; Wynn, 1992, 1998).

While recent research has highlighted the importance of the systems for parallel individuation and set-based quantification for acquisition of the counting principles (e.g., Carey, 2001, 2004; Carey & Sarnecka, 2006; Condry & Spelke, 2008; Le Corre & Carey, 2007; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007), the present work argues for a role of the analog magnitude system as well (see also Barth, 2008). Indeed, the confusability of the analog representations may help explain the protracted course of counting acquisition in early childhood. If, for example, the analog magnitude representations are involved in the child's early representations of 'three' or 'four' and those analog representations are highly overlapping, this could present a major challenge for the child.

In addition, children with inherently greater precision in their analog systems might be expected to acquire the cardinality principle earlier and more easily, which may in turn lead to advantages in later mathematical achievements. Indeed, Halberda and colleagues (Halberda, Mazzocco, & Feigenson, 2008) recently found that an individual's skill with the approximate number system was linked to skills in symbolic math, independent of other cognitive and performance factors. Specifically, individual differences in the precision of the analog system at age 14 were reliably correlated to standardized assessments of symbolic mathematics measured from kindergarten through sixth grade (Halberda et al., 2008).

Further, work with atypically-developing populations also posits an important role for the analog magnitude system in the acquisition of more complex numerical concepts (e.g., Van Herwegen, Ansari, Xu, & Karmiloff-Smith, 2008). Van Herwegen et al. (2008) looked at both the object-tracking system and the analog magnitude system in infants and toddlers with Williams syndrome (WS), a condition associated with significant numerical deficits spanning both verbal counting and non-verbal representations of quantity in older WS individuals (e.g., Paterson, Girelli, Butterworth, & Karmiloff-Smith, 2006). Infants with WS successfully differentiated between 2 and 3 objects but failed to detect the difference between 8 and 16. This pattern of success with the object-individuation system and difficulty with the analog magnitude system, combined with deficits with more advanced numerical abilities led the authors to conclude that "…individual differences in large number processing in infancy are more likely than small number processing to be predictive of later development of numerical cognition" (Van Herwegen et al., 2008). The present studies also argue for the role of individual differences in the large number processing system (i.e., the analog magnitude system) as children are mastering more advanced numerical concepts.

While the present studies find the strongest evidence to date for the presence of analog magnitude representations in early mappings of large numbers in children's count list *before* the acquisition of the cardinality principle, future work will be useful in outlining how these numerical skills interact. For example, intervention studies that train either analog skills (e.g., quantity discrimination and estimation) or verbal counting skills while assessing the other can provide important insight into the causal relations between these representational systems. With this work, researchers can move towards a more coherent account of the representations underlying acquisition of the counting principles and other mathematical abilities.

Appendix A

The stimuli and scenarios used in Study 2 are described in Table A.

Appendix B

Individual mean, standard deviation, and COV graphs for the 2, 3, and 4-proficiency groups are shown in Fig. B. Individual analyses for these groups are shown in Table B. Due to the small n for proficiency level 1, this group is not included.

Appendix C

Figs. C1–C3 show Study 2 analyses of mean, standard deviation, and COV including trials where children responded with "all" items. These three graphs are parallel to Figs. 5–7 and present findings from 28 children at proficiency levels 1–4. The inclusion of "all" trials allowed for the addition of four subjects who were otherwise eliminated from analyses for insufficient trials at each of the ten numbers requested.

Table A
Stimuli and scenarios used in Study 2.

Type of object	Total number	Scenario given
Plastic fish	30	"This is a pond (i.e., a plastic box) where these fish like to go swimming. Can you put *x* fish in the pond?"
Strawberry erasers	30	"Let's go to the zoo and feed the animals (i.e., plastic boxes with pictures of different animals inside). Can you give the [animal] *x* strawberries?
Rubber balls	30	"Now that the animals have eaten, they want to play. Can you give the [animal] *x* balls to play with?"
Plastic frogs	30	"This is a lake where these frogs like to go swimming. Can you put *x* frogs in the lake?"
Gold coins	144	"We are going to go shopping for stickers. Can you give me *x* coins for a new sticker?"
Plastic sharks	27	"This is the ocean where these sharks like to go diving. Can you put *x* sharks in the water?"

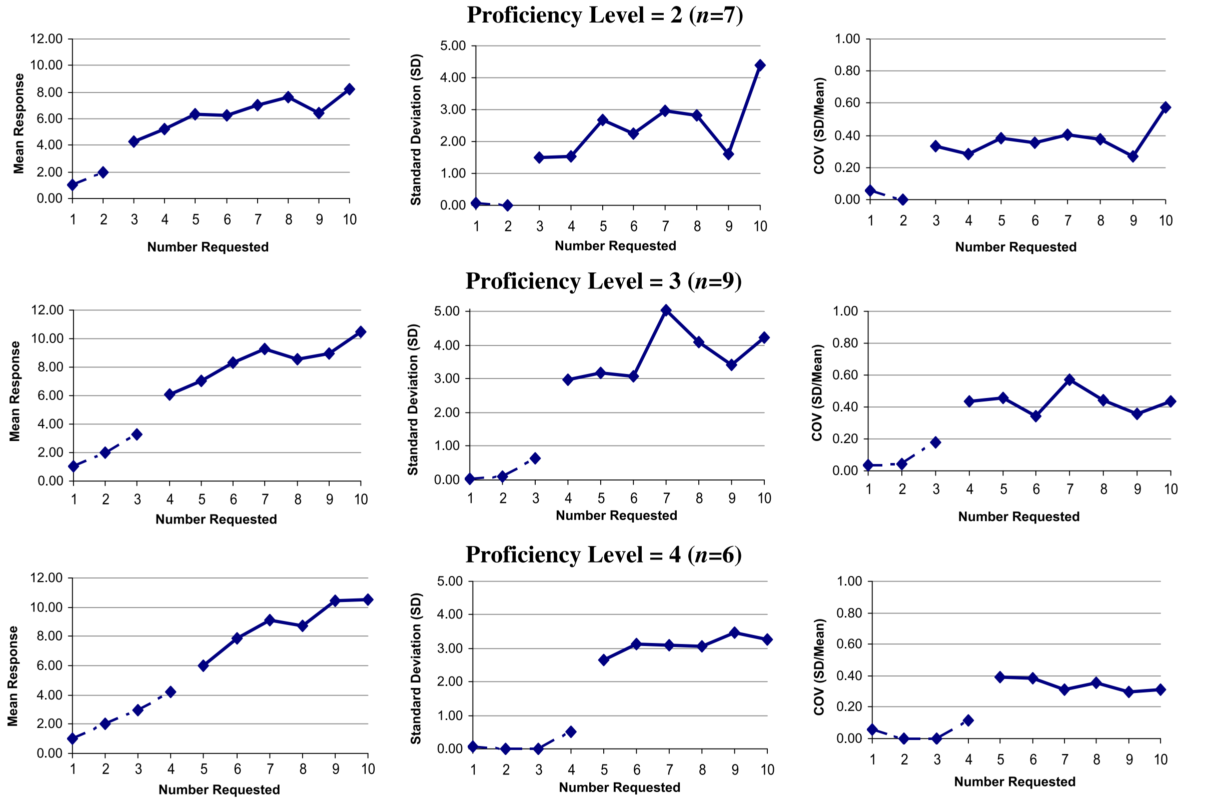

Fig. B. Performance in Study 2 by preschoolers at proficiency levels 2, 3, and 4. Graphs on the left illustrate average number of items given for numbers 1–10. Graphs in the middle illustrate the standard deviation (*SD*) for responses given for numbers 1–10. Graphs on the right illustrate COV (*SD*/Mean) values. Dotted lines represent mean response and SD at or below the proficiency level, *p*; solid lines represent values beyond *p*. When an analog magnitude system is used for estimation, mean response and standard deviation should grow proportionally as the number estimated increases, resulting in a flat COV for values beyond *p*.

Table B
Slopes for mean response, standard deviation (*SD*), and COV (*SD*/mean) for values beyond proficiency level, *p*, for 2-, 3-, and 4-proficiency groups.

	Slope	*t*	*df*	*p*
2-proficient				
Mean response	0.46 (*0.73*)	1.64	6.00	0.15
Standard deviation	0.26 (*0.24*)	2.83	6.00	0.03*
COV	0.02 (*0.03*)	1.52	6.00	0.42
3-proficient				
Mean response	0.61 (*0.69*)	2.66	8.00	0.03*
Standard deviation	0.18 (*0.40*)	1.38	8.00	0.21
COV	−0.004 (*0.04*)	−0.31	8.00	0.76
4-proficient				
Mean response	0.86 (*0.96*)	2.19	5.00	0.08
Standard deviation	0.12 (*0.61*)	0.48	5.00	0.65
COV	−0.02 (*0.04*)	−1.05	5.00	0.34

Note: These are the results of two-tailed one-sample *t*-tests of each slope against 0. Slopes for Mean and SD are predicted to be positive and significantly different from 0, while slopes for COV are predicted to be no different from 0.

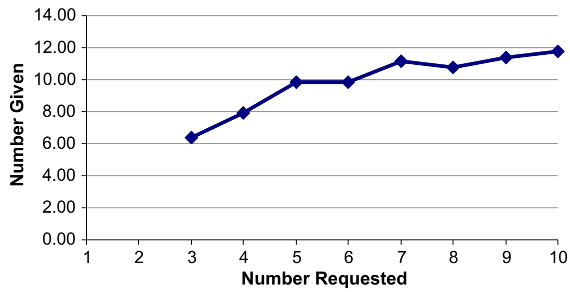

Fig. C1. Mean number of items given beyond a child's proficiency level for requests of 3–10 items. The average slope of the mean response for items beyond a child's proficiency level is 0.59 (*0.95*) and significantly different from 0, $t(27) = 3.285$, $p = .003$. This finding is consistent with that reported in Fig. 5.

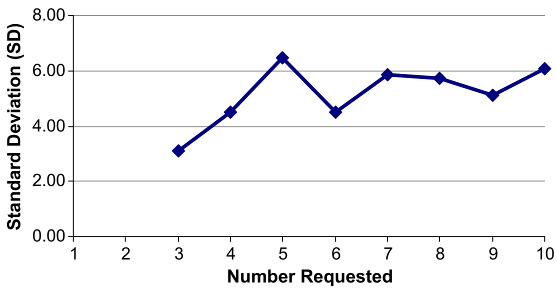

Fig. C2. Standard deviation of responses given beyond a child's proficiency level for requests of 3–10 items. The average slope of the mean response for items beyond a child's proficiency level is 0.07 (*0.61*), a value that is not significantly different from 0, $t(27) = 0.62$, $p = 0.5$. Although the slopes reported here and in Fig. 6 are both greater than 0, when "all" responses are included, this value is no longer significantly different from 0.

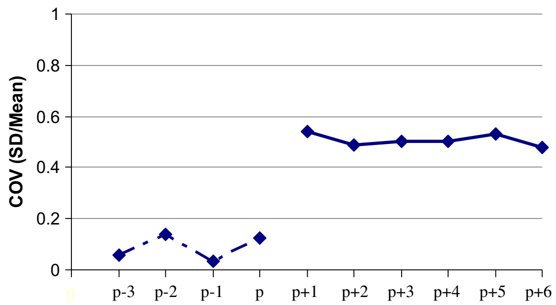

Fig. C3. Coefficient of variation values collapsed across proficiency levels (p) of 1, 2, 3, and 4 for values from $p - 3$ through $p + 6$. The COV values beyond p show a slope no different from 0, $M = -.006$, $SD = .05$, $t(27) = -.633$, $p = 0.5$. This finding is consistent with that reported in Fig. 7.

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